Minimum energy per bit with and without feedback

With feedback: convergence is very fast.

• Without feedback: convergence to Shannon limit is slow, \( O(\frac{1}{\sqrt{n}}) \)

• With feedback: convergence is very fast.

• Surprisingly: decision feedback is enough!

• Plot: block error probability \( \epsilon = 10^{-3} \), see [PPV10a]

• Our work: How do \( M^*(E, \epsilon) \) and \( M^f(E, \epsilon) \) compare for finite \( E \)?

### Results

**Highlights:**
- Horizontal axis: \( k = \log_2 M \)
- Without feedback: convergence to Shannon limit is slow, \( O(\frac{1}{\sqrt{n}}) \)
- With feedback: convergence is very fast.
- Surprisingly: decision feedback is enough!
- Plot: block error probability \( \epsilon = 10^{-3} \), see [PPV10a]

• Without feedback:
  \[
  \log M^*(E, \epsilon) \approx \frac{E}{N_0} \log \epsilon - \frac{E}{N_0} Q^{-1}(\epsilon) + \frac{1}{2} \log \frac{E}{N_0} + O(1), \quad \epsilon > 0
  \]

  \[
  \log M^f(E, \epsilon) = 0
  \]

• With feedback:
  \[
  \log M^f(E, \epsilon) \approx \frac{E}{N_0} \log \epsilon - \frac{E}{N_0} Q^{-1}(\epsilon) + O(\log \epsilon), \quad \epsilon > 0
  \]

**Note:** feedback makes zero-error communication possible.