Energy efficient random access for the quasi-static fading MAC

Suhas S Kowshik
MIT
suhask@mit.edu

Kirill Andreev
SkolTech (Moscow)
k.andreev@skoltech.ru

Alexey Frolov
SkolTech (Moscow)
al.frolov@skoltech.ru

Yury Polyanskiy
MIT
yp@mit.edu

***Abstract*—We discuss the problem of designing channel access architectures for enabling fast, low-latency, grant-free and uncoordinated uplink for densely packed wireless nodes. Specifically, we extend the concept of random-access code introduced at ISIT'2017 by one of the authors to the practically more relevant case of the AWGN multiple-access channel (MAC) subject to Rayleigh fading, unknown to the decoder. We derive bounds on the fundamental limits of random-access coding and propose an alternating belief-propagation scheme as a candidate practical solution. The latter's performance was found to be surprisingly close to the information-theoretic bounds. It is curious, thus, that while fading significantly increases the minimal required energy-per-bit E_b/N_0 (from about 0-2 dB to about 8-11 dB), it appears that it is much easier to attain the optimal performance over the fading channel with a practical scheme by leveraging the inherent randomization introduced by the channel. Finally, we mention that while a number of candidate solutions (MUSA, SCMA, RSMA, etc.) are being discussed for the 5G, the information-theoretic analysis and benchmarking has not been attempted before (in part due to lack of common random-access model). Our work may be seen as a step towards unifying performance comparisons of these methods.**

I. Introduction

Presently, wireless networks are starting to see a new type of load (a so called mMTC or machine-type communication), in which hundreds of thousands of devices are serviced by a single base station, each communicating very small and infrequent data payloads. In the interest of reducing hardware complexity, reducing latency and improving energy consumption, the conceptual paradigm shift is to move to the *grant-free* access management, in which uplink communication is not orthogonalized by the base-station (as is done in today's systems). This requires new kind of codes that can be decoded from uncoordinated and colliding transmissions.

An information theoretic formulation of this problem was done in [1] where the author considered an additive white Gaussian noise (AWGN) random access channel (RAC) model. In this formulation the random access is modeled as follows: each of K_a active users encodes his k-bit message into an n-symbol codeword. The receiver observes superposition of K_a codewords corrupted by the AWGN. There are a number of challenges in this model: finite blocklength (FBL) effects due to small payload size, massive number of users (comparable to blocklength), sparsity due to random access and incorporating accurate channel models. However, the most crucial departure from canonical MAC is that the users are required to share the same codebook (i.e. they are unindentifiable, unless they desire to put their identity as part of the k-bit payload), and the decoder is only required to provide an unordered list of user messages. In the follow-up works, this problem has also been called *unsourced random access* [2–4]. Another important aspect of this new formulation is the notion of per-user probability of error (PUPE) which is defined as the average (over the active users) fraction of the transmitted messages that are misdecoded. (Recall that classical definition declares error even if any one of the messages decoded incorrectly.)

A brief summary of the AWGN-RAC work is as follows: [1] provided bounds on fundamental limits and identified tremendous energy-inefficiency of existing schemes (like slotted ALOHA [5] which is used for the LTE PRACH channel [6]). In [2, 3, 7–10] various (progressively better) practical schemes were proposed, with the latest being within a few dB of the fundamental limits. In [4] the approach of [3] was extended to quasi-static MIMO fading RAC.

The goal of this paper is to study the more practically relevant model of quasi-static fading AWGN-RAC in which different user's transmissions are attenuated by random fading gains, unknown to the receiver. We will discuss both the fundamental limits and a practical low-complexity LDPC scheme with surprisingly good performance.

The structure of the paper is as follows. In section II, we state the definition of the random-access code and formally define the system model; in section III, we state our main achievability result and discuss T-fold ALOHA from [7]; in section IV, a converse bound is presented based on the meta-converse theorem from [11]; in section V, the low complexity iterative decoding scheme is discussed along with the alternating belief propagation decoder; in section VI, numerical results are presented along with possible future research directions.

II. System Model

We follow the definition of a code from [1]. Fix an integer $K_a \geq 1$ – the number of active users. We consider the single antenna quasi-static fading MACs:

$$Y^n = \sum_{i=1}^{K_a} H_i X_i^n + Z^n \tag{1}$$

where $Z^n \sim \mathcal{CN}(0, I_n)$, and $H_i \overset{iid}{\sim} \mathcal{CN}(0,1)$ are the fading coefficients which are independent of $\{X_i^n\}$ and Z^n. We call this the K_a–MAC. We assume that there is a maximum power constraint on the inputs:

$$\|X_i\|^2 \leq nP \tag{2}$$

This material is based upon work supported by the National Science Foundation under Grant No CCF-17-17842, CAREER award under grant agreement CCF-12-53205, the Hong Kong Innovation and Technology Fund (ITS/066/17FP) under the HKUST-MIT Research Alliance Consortium and a grant from Skoltech–MIT Joint Next Generation Program (NGP).

The research of A. Frolov and K. Andreev was supported by the Russian Science Foundation (project no. 18-19-00673).

Definition 1 ([1]). *An (M, n, ϵ) random-access code for the MAC* (1) *is a pair of (possibly randomized) maps $f : [M] \to \mathbb{C}^n$ (the encoder) and $g : \mathbb{C}^n \to \binom{[M]}{K_a}$ such that if $W_1, ..., W_{K_a}$ are chosen independently and uniformly from $[M]$ and $X_j = f(W_j)$ then the average (per-user) probability of error (PUPE) satisfies*

$$P_e = \frac{1}{K_a} \sum_{j=1}^{K_a} \mathbb{P}\left[E_j\right] \leq \epsilon \tag{3}$$

where $E_j \triangleq \{W_j \notin g(Y^n)\} \cup \{W_j = W_i \text{ for some } i \neq j\}$ and Y is the channel output.

From this definition, all users use the same codebook and the receiver outputs a list of K_a codewords. We emphasize that there can be potentially an unbounded number of users, but only K_a of them are active. If each user has a message of size k and transmits at power P per symbol, then the energy-per-bit is given by $E_b/N_0 = \frac{nP}{k}$. Lastly we assume that the knowledge of channel gain realizations is unknown at both transmitters and receiver. In the rest of the paper we drop the superscript n unless it is unclear.

III. K_a–MAC FBL ACHIEVABILITY BOUNDS

A. T-fold ALOHA [7]

In this section, we discuss our main achievability bound based on T–fold ALOHA protocol [7]. The idea is the following. Let $T, n_1 \in \mathbb{N}$ be such that $T < K_a$ and $n_1 < n$. The time slot or frame of length n is partitioned into $L = n/n_1$ subframes of length n_1. The common codebook is of blocklength n_1. Each user chooses a slot to send his message uniformly at random independently of other users. Suppose there is a code such that if there are at most T users transmitting in a given block, the decoder can decode all the messages but if more than T users are transmitting then nothing is decoded. With this protocol, we have the following achievability bound on PUPE.

$$\epsilon_T \leq 1 - \sum_{r=1}^{T} \bar{P}_e(M, n_1, r, LP) \binom{K_a - 1}{r - 1} \left(\frac{1}{L}\right)^{r-1} \left(1 - \frac{1}{L}\right)^{K_a - r} \tag{4}$$

where $1 - \bar{P}_e(M, n_1, r, P) = P_e(M, n_1, r, P) = \epsilon$ of an (M, n_1, ϵ) code used over the r active user fading RAC with power constraint P.

For evaluations, we will use (4) with actual simulated numbers for P_e from an LDPC code, or we will use $P_{e,genie}$ from a random coding bound presented in (5) in the next subsection. We note here that, to use (5) in (4), a genie needs to inform the decoder about the active number if users in a subframe. Hence $P_{e,genie}$ is not a true bound and is plotted only for reference. In the non-fading (AWGN) setting [7] the problem of infering the number of active users in subframe can be easily resolved by simply measuring the average energy of the signal. This method is not applicable in the presence of fading – see section VI for more. Also note that LDPC-based scheme performs true blind decoding, i.e. it determines the number of users in a slot and decodes the messages. So we do not assume the knowledge of the number of users in a slot in this case.

B. FBL Achievability bounds

Here, we present a random coding achievability bound for the channel when the decoder knows the number of active users. So, in the T–fold ALOHA setting, this would be a *genie*–aided FBL bound since a genie provides provides the decoder with the number of active users in each slot. We set up some notation. If $C \subset \mathbb{C}^n$ then denote P_C as the orthogonal projection operator onto the subspace spanned by C.

Theorem III.1. *Fix $P' < P$. Let $K_1 \leq K_2$. Then there exists an (M, n, ϵ) (with $\epsilon \geq \frac{K_2 - K_1}{K_2}$) random access code for the K_2–MAC* (1) *satisfying power constraint P (see* (2)*) and*

$$\epsilon \leq \frac{K_2 - K_1}{K_2} + \frac{1}{K_2} \sum_{t=1}^{K_1} K_{1,t} p_t + p_0 \tag{5}$$

with $p_0 = \frac{\binom{K_2}{2}}{M} + K_2 \mathbb{P}\left[\frac{P'}{2} \sum_{i \in [2n]} W_i^2 > nP\right]$, $W_i \overset{iid}{\sim} \mathcal{N}(0, 1)$ and

$$p_t \leq \inf_{\delta > 0} \left(\binom{K_2}{K_{1,t}} e^{-(n - K_1)\delta} + \mathbb{P}\left[\bigcup_{\substack{S_0 \subset [K_2] \\ |S_0| = K_{1,t}}} \{G(Y, S_0, c_{S_0}, t) \geq V_{n,t}\} \right] \right) \tag{6}$$

where

$$G(Y, S_0, c_{S_0}, t) = \frac{\|Y\|^2 - \max_{\substack{S_2 \subset S_0 \\ |S_2| = t}} \left\| P_{c_{[S_2 \cup ([K_2] \setminus S_0)]}} Y \right\|^2}{\|Y\|^2 - \left\| P_{c_{[[K_2] \setminus S_0]}} Y \right\|^2} \tag{7}$$

$$K_{1,t} = K_2 - K_1 + t \tag{8}$$

$$V_{n,t} = e^{-\left(\delta + \frac{\ln\binom{M - K_2}{t}}{n - K_1} + \frac{\ln\binom{n' - 1}{t - 1}}{n - K_1}\right)} \tag{9}$$

$$n' = n - K_1 + t \tag{10}$$

and, $\mathcal{C} = \{c_i : i \in [M]\}$ denotes the Gaussian codebook, $\{c_i : i \in [K_2]\}$ are the transmitted codewords, $c_S = \{c_i : i \in S\}$ and Y is the received vector.

Proof idea: The idea is to use random coding using Gaussian codebook. For decoding, we use a projection based decoder inspired from [12]. The idea is that the received vector will lie in the subspace spanned by the sent codewords in the absence of additive noise. Formally, fix an output list size K_1. Let $\mathcal{C}$ denote the common codebook. Then, upon receiving Y from the channel, the decoder outputs a list of K_1 codewords which form a subspace, such that projection of Y onto this subspace is maximum i.e., it outputs $g(Y)$ given by

$$g(Y) = \{f^{-1}(c) : c \in \hat{C}\}$$
$$\hat{C} = \arg \max_{C \subset \mathcal{C} : |C| = K_1} \|P_C Y\|^2 \tag{11}$$

where f is the encoding function.

The projection decoding is also called nearest-subspace decoding, and has been used in the compressed sensing literature [13, 14]. One might prefer to view it as a kind of maximum likelihood (ML) decoding as well for the likelihood $P_{Y|X,H}(y, \{x_i\}, \{h_i\}) = \frac{1}{\pi^n} e^{-\left\|y - \sum_i h_i x_i\right\|^2}$.

Further projection decoding achieves ϵ–capacity of the vanilla K_a–user quasi static fading MAC (with different codebook and the usual joint probability of error) [15, 16]. ■

We make the following observation about K_1. When K_2 is large, it is hard to decode messages of users with the least $|H_i|^2$ since its expectation is $\frac{1}{K_2}$. So, as observed in [17], it makes sense to drop a fraction of users which have low channel gains and decode the rest.

As discussed in section VI below, the computation of the bound in the theorem is done using MCMC methods for small values of K_2. Further, this bound holds even when using spherical codebook i.e., codewords distributed uniformly on the (complex) power shell (with $p_0 = \frac{\binom{K_2}{2}}{M}$). For larger values of K_2, a computable bound is presented in [18].

IV. Converse bound

In this section we describe a simple converse bound based on results from [12] and the meta-converse from [19].

Theorem IV.1. *Let*

$$L_n = n\log(1+PG) + \sum_{i=1}^{n}\left(1 - |\sqrt{PG}Z_i - \sqrt{1+PG}|^2\right) \quad (12)$$

$$S_n = n\log(1+PG) + \sum_{i=1}^{n}\left(1 - \frac{|\sqrt{PG}Z_i - 1|^2}{1+PG}\right) \quad (13)$$

where $G = \|H\|^2$ and $Z_i \overset{iid}{\sim} \mathcal{CN}(0,1)$. Then for every n and $0 < \epsilon < 1$, any $(M, n-1, \epsilon)$ code for the quasi static K_a MAC satisfies

$$\log(M) \le \log(K_a) + \log\frac{1}{\mathbb{P}\left[L_n \ge n\gamma_n\right]} \quad (14)$$

where γ_n is the solution of

$$\mathbb{P}\left[S_n \le n\gamma_n\right] = \epsilon. \quad (15)$$

V. Low-complexity iterative coding scheme

In this section we present a low-complexity iterative coding scheme based on LDPC codes, which allows one to decode user messages in a slot.

Recall, that the users utilize the same codebook. Let us denote it by $\mathcal{C}$ and explain how to construct it. We start with a binary $[n,k]$ LDPC codebook and replace each 0 with $+\sqrt{P}$ and each 1 with $-\sqrt{P}$. Let us show the bit-wise MAP decoding rule for the j-th bit of the i-th user below

$$\hat{X}_{i,j} = \arg\max_{X_{i,j}\in\pm\sqrt{P}} \mathbb{E}\left[\sum_{\sim X_{i,j}} p_{Y|X}\left(Y \mid \sum_{k=1}^{T} H_k X_k\right) \prod_{k=1}^{T} \mathbb{1}_{X_k\in\mathcal{C}}\right] \quad (16)$$

where the expectation is taken over $H_1, H_2, \ldots, H_T$. Following [20], the summation "$\sim X_{i,j}$" means that we sum over all positions in all user codewords, except $X_{i,j}$.

The aim of the decoder is to recover all the codewords based on the received vector Y. The decoder employs a low-complexity iterative belief propagation (BP) decoder that deals with a received soft information presented in log likelihood ratio (LLR) form. The decoding system can be represented as a graph (factor graph, [21]), which is shown in Fig. 1.

There are four types of nodes in the graph. User LDPC codes are presented with use of Tanner graphs with variable (red color) and check nodes (blue color). At the same time there is a third kind of nodes in the figure – functional nodes (marked with green color). These nodes correspond to the elements of the received sequence Y. The fourth kind of nodes (magenta nodes) correspond to fading coefficients. We note, that the decoder also performs an estimation of fading coefficients (latent variables for our decoder).

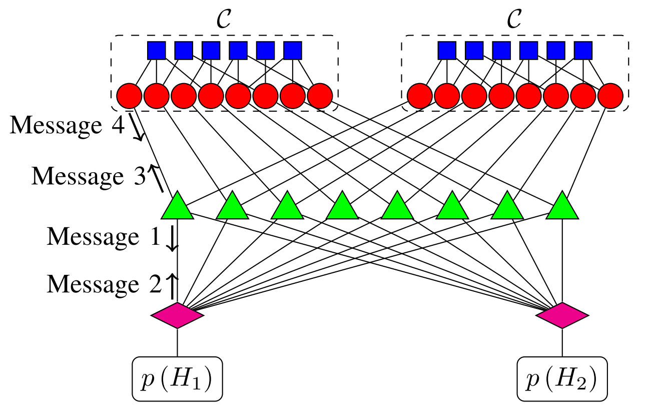

Fig. 1: Iterative joint decoding algorithm (alternating BP-decoder), factor graph

The decoding algorithm is based on the iterative message passing procedure. There are two types of iterations in our system: inner iterations, which are used for LDPC code decoding and outer iterations used for fading coefficients estimation. In what follows we mean an outer iteration in all the cases where the type of iteration is not specified. Within the iterations we decode all the users in a sequential manner. Let us consider a single user decoding. This process consists of calculation and passing of four message types marked by arrows and number in Fig. 1. We note, that both fading coefficients and LLRs for another users remain fixed during this process. Every message is described in details below:

a) Message type 1 (from functional nodes to fading nodes): Without loss of generality let us consider the first functional node. Assume we received a symbol y. By $x_i = X_{i,1} \in \{+\sqrt{P}, -\sqrt{P}\}$, $i = 1,\ldots,T$, we denote symbols sent by the users. Let us show how to calculate a posterior pdf of H_1 from the first functional node. We denote this message by $R_1^{(1)}$ and calculate it as follows

$$R_1^{(1)}(h_1) \propto \mathbb{E}\left[\sum_{x_1,x_2,\ldots x_T} p(y|\sum_{j=1}^{T} H_j x_j)\prod_{j=2}^{T}\Pr(x_j)\right], \quad (17)$$

where the expectations are taken over $H_2,\ldots,H_T$. Such updates are calculated at every functional node and denoted by $R_1^{(i)}$, $i = 1,\ldots,n$. In order to construct the practical implementation one needs an efficient method of fading coefficients pdf approximation. This can be done by means of Gaussian mixtures [18].

b) Message type 2 (from fading nodes to functional nodes): We denote the message from j-th fading node to i-th functional node by $Q_j^{(i)}$, this message is a pdf. In order to find it we need to calculate the product of incoming messages. Let us consider a message from the first fading to the first functional node, we have

$$Q_1^{(1)}(h_1) = \prod_{i=1}^{n} R_1^{(i)}(h_1), \quad (18)$$

In a conventional message passing algorithm the outgoing message is calculated based on messages which come through all the edges except its own edge. But here to reduce the complexity we approximate the complicated message update at fading nodes via the product of a few randomly selected incoming messages.

c) Message type 3 (from functional nodes to LDPC codes): Again let us consider the first functional node. Assume we received a symbol y. By $x_i = X_{i,1} \in \{+\sqrt{P}, -\sqrt{P}\}$, $i = 1, \dots, T$, we denote symbols sent by the users. Let us note, that a posterior LLR for x_1 can be calculated as follows

$$L(x_1) = \log \frac{\mathbb{E}\left[\sum\limits_{x_1=+\sqrt{P}, x_2, \dots x_T} p(y|\sum\limits_{j=1}^{T} H_j x_j) \prod\limits_{j=2}^{T} \Pr(x_j)\right]}{\mathbb{E}\left[\sum\limits_{x_1=-\sqrt{P}, x_2, \dots x_T} p(y|\sum\limits_{j=1}^{T} H_j x_j) \prod\limits_{j=2}^{T} \Pr(x_j)\right]},$$

where the expectations are taken over $H_1, H_2, \dots, H_T$ and $p(y|a) = \frac{1}{\sqrt{\pi}} \exp(-(y-a)^2)$. Note, that for practical implementation the Monte-Carlo sampling method can be used for expectations.

d) Message type 4 (LDPC decoding): After functional nodes decoding one needs to upgrade the LLR for given user with LDPC iterative decoder. This part is standard, i.e. each user utilizes standard BP decoding algorithm (Sum-Product or Min-Sum, [20]) to decode an LDPC code.

As soon as the iterative decoder operates as an optimization task which is split between two groups of variables (user LLRs and fading coefficients), one can expect this algorithm to converge to some local maximum of (16). This can be a source of error floor and this error floor was observed during numerical experiments. To overcome this problem one can start the decoding algorithm multiple times and handle functional nodes in random order at every decoding iteration. As soon as GMs are merged and pruned, this provides some source of randomness and pushes the decoding procedure to possibly different local maxima. This approach eliminates the error floor problem and allows another opportunity – **blind user decoding**, i.e. we determine the number of active users in a slot and recover their messages. Let us give a short description. Given multiple decoding attempts, one can select a set of unique codewords that were successfully decoded (have zero syndrome). Every attempt can detect different codewords combinations. The final output of the decoder is the union of such sets. We note that the approach presented here is similar to the approach from [22]. Nevertheless, the main differences are: a) we consider same codebook case, b) we perform blind user decoding.

VI. Numerical results and discussion

In this section we present the plots of the minimum energy per bit required to achieve a probability of error $\epsilon = 0.1$ as a function of K_a for the channel (1). Fig. 2 shows plots of various schemes. The parameters used for evaluation are blocklength $n = 30000$ and message size $k = 100$ bits. Note that we assume the fading coefficients to remain constant only within the slot of length n_1. Next we describe how each of these curves was obtained.

For T-fold ALOHA using FBL+genie bound, we use the bound for p_t given in (6). For each K_a we find the optimum L (as an optimization over both L and P) that minimizes E_b/N_0 such that ϵ_T in (4) is less than 0.1. For computational reasons, we approximate PUPE for the fading channel by the sum capacity [11]. Then we use the spherical codebook i.e., codewords uniformly and independently sampled from the (complex) power shell in dimension $n_1 = \lfloor n/L \rfloor$ to compute the probability of error according to (4) where $P_e(M, n_1, r, LP)$ is computed using brute-force Monte-Carlo simulation of (6) with the choice $K_1 = K_2 = r$. Since $r \leq T$ is small it would not make sense to drop an user. To this end, we produce 2000 samples, from which we construct the kernel density approximation of the cumulative distributive function (CDF) of the statistic $\max_{\substack{S_0 \subset [r] \\ |S_0| = t}} G(Y, S_0, c_{S_0}, t)$ for each $t \leq r$. Then this smooth approximation is used to optimize over δ in (4).

For T-fold ALOHA using the iterative coding scheme, we have used (n_1, k) LDPC codes with $k = 100$ and blocklength $n_1 \in \{200, 400\}$. We note, that two codes are enough to cover the interval $1 \leq K_a \leq 250$. For each of these codes, we get PUPE vs E_b/N_0 curves and choose the best code (the best code requires the smallest E_b/N_0 in order to achieve PUPE $\leq \epsilon = 0.1$) for each value of K_a. The best waterfall curves for the different number of users are presented in Fig. 3. Note again, that in LDPC-based scheme we perform honest blind slot decoding (without assuming the knowledge of user count in a slot). It can be seen from Fig. 2 that the performance of T–fold ALOHA for iterative decoding scheme is very close to that of T–fold ALOHA with random coding bounds for small K_a. The gap increases with K_a because of our limited choices of LDPC codes, i.e. due to BPSK modulation, we are constrained by $n_1 \geq k$.

We have also plotted the result of treat interference as noise (TIN) decoding. Here we have used standard second order capacity approximation [11]. It is easy to get an actual random coding bound for TIN similar to theorem III.1, but we don't expect it to be better.

Also presented for reference is the Shamai-Bettesh capacity bound from [17]. It is an asymptotic bound ($n \to \infty$) for the probability of error per-user in the case of symmetric rate and large K_a. It, however, applies to the case of different user codebooks and full knowledge of the CSI at the decoder. (The decoder decodes the largest set of users whose rate tuple vector is inside the corresponding (instantaneous) capacity region by considering the dropped users as additive noise. The ratio of this largest set with the total number of users determines the per-user error.)

We have also plotted performance of optimal decoder which requires the computation of true posteriors. This is computed using results in the asymptotic regime [18, 23] where K_a and M scale linearly with blocklength. Although asymptotic, we suspect this predicts the FBL performance quite well since it was indeed the case in the AWGN setting as shown in [1, 24]. The converse from (14) and (15) is also plotted. This is in essence a single user based converse bound. A Fano type converse is possible but it is worse than this bound for the parameter range we work with. The converse presented here illustrates the fact the E_b/N_0 requirements are necessarily higher compared to the AWGN channel in [1]. See [18] for more plots and details.

In all, we observe that even at $K_a = 100$, the random coding based 4–fold ALOHA performance is off from the converse bounds (and asymptotic $n \to \infty$ predictions from Shamai-Bettesh or replica method) by only a few dB. This is in dramatic contrast with currently dominant schemes treating interference as noise or employing a (slotted) ALOHA.

In terms of future work, one of the most important things is to relax the assumption on the knowledge of the number of users in a slot in T–fold ALOHA, for instance, by changing the RAC model to account for unknown but bounded number of users (similar to support recovery with unknown sparsity in compressed sensing). Another important factor is frame-synchronization which we have assumed. It is interesting to

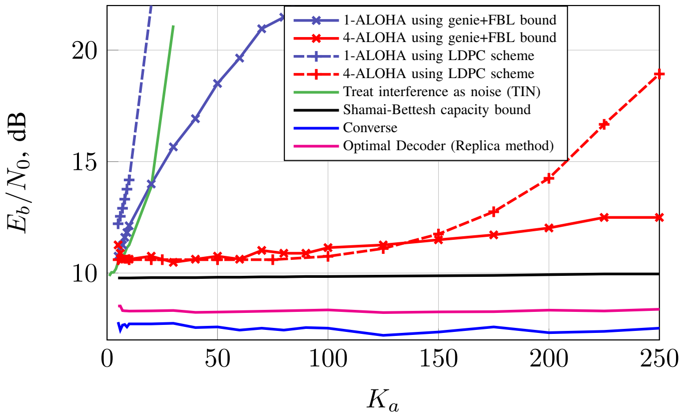

Fig. 2: K_a vs E_b/N_0 for $\epsilon \leq 0.1$, $n = 30000$, $k = 100$ bits

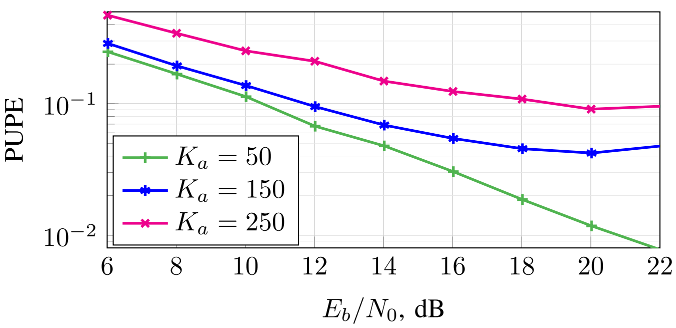

Fig. 3: E_b/N_0 vs PUPE for $n = 30000$, $k = 100$ bits

see how the performance is affected when frame synchrony is lost. In terms of bettering the performance, it is necessary to consider a MIMO scenario with the receiver having multiple antennas. We leave these to future work.

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