Abstract—The problem of unsourced random access in the asynchronous quasi-static Rayleigh fading channel is considered. A transmission scheme operating in the frequency domain is proposed. This trick allows to convert the synchronization problem to the phase shift estimation problem. The decoder utilizes treat interference as noise paired with the successive interference cancellation (TIN-SIC) approach. We present complete simulation results for the new decoder. As we are interested in energy efficiency we provide energy-per-bit vs number of active users curves and show this decoder to outperform the joint decoder from [1, 2] for both synchronous and asynchronous cases.

I. INTRODUCTION

The main goal of current wireless networks is to handle user-generated traffic and thus increase the spectral efficiency. Indeed, this parameter is of most importance when we speak about ultra-high-definition (UHD) video streaming. At the same time, next-generation wireless networks face the problem of massive machine-type communications (mMTC) with up to million devices connected to a single base station. The traffic of the devices is significantly different from the traffic generated by human-users and consists of short packets that are sent sporadically. The main goal is not to increase the spectral efficiency but to provide connectivity and energy efficiency as we do not plan to exchange millions of batteries every day. Current transmission schemes are highly inefficient in this regime. The problem of massive random access is of critical importance for 5G/6G and Internet of Things (IoT) applications and is being actively discussed within 3GPP standardization committee. Several approaches [3–6] to fit new traffic scenarios were proposed, but the lack of implementation details does not allow us to judge how good these schemes are.

The massive random access problem was formalized in [7], where the finite length random coding bound for the Gaussian multiple access channel (MAC) was derived. The main idea of [7] is to split identification and decoding tasks. Users (devices) are assumed to utilize the same encoder and the decoder task is to recover only the set (list) of transmitted messages without user identities. Indeed, this idea is very promising for massive random access, when the number of devices is extremely large and it is difficult to create different encoders for the users. Since the users utilize the same encoder and it is not possible to determine the source of the message this problem is also known as unsourced random access in the literature. The improvement of the random coding bound for the Gaussian MAC was given in [8]. There is plenty of paper with low-complexity coding schemes for the Gaussian MAC. Here we mention the main approaches:

- $T$-fold\textsuperscript{1} slotted ALOHA (SA) in combination with compute-and-forward strategy [9, 10];
- $T$-fold irregular repetition slotted ALOHA (IRSA) in combination with LDPC codes [11–13];
- $T$-fold IRSA in combination with polar codes [14];
- Coupled compressive sensing [15, 16];
- Sparse regression codes [17];
- Sparse spreading or joint decoding graph [18];
- Polar codes with random spreading and correlation-based energy detector [19];
- The case of correlated messages from devices [20].

Gaussian MAC is an ideal channel model. To be closer to the reality the authors of [1, 2, 21, 22] considered the synchronous quasi-static Rayleigh fading MAC and derived asymptotic and finite-length achievability/converse bounds as well as a low-complexity coding scheme utilizing carefully constructed LDPC codes and joint message passing decoder. The performance of the proposed scheme was found to be very close to the achievability bound for up to 150 active users. It is important to note that the authors assumed the absence of channel state information (CSI) both at the transmitters (we want the device to be as simple as possible) and the receiver (it seems to be impossible to estimate the channel for a million of devices especially for short sporadic packets). In [23] a MIMO channel was considered and an efficient scheme was proposed. The authors assume multiple antennas to present at the receiver only.

It is clear that completely synchronous scenario is not feasible in any practical implementation, especially for the

\textsuperscript{1} by $T$-fold ALOHA we mean ALOHA with multi-slot reception, i.e. we design slot codes to recover collisions of order up to $T$. Classical slotted ALOHA corresponds to the case $T = 1$. 

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case of simple low-power devices. The classical \(^2\) asynchronous MAC was widely studied in the literature, see e.g. a paper [24] in which a capacity region of asynchronous Gaussian MAC was derived. We are interested in asynchronous unsourced random access. There are only a few papers devoted to this topic also known under the name asynchronous neighbor discovery [25, 26] which are based on compressive sensing approach.

In this paper we extend our methods for the case of unsourced asynchronous fading MAC. We continue the research started in [27] where a decoder operating in the frequency domain was proposed. By this trick the synchronization problem was converted to the phase shift estimation problem. The joint decoder from [1, 2] appeared to be unstable for the asynchronous case of simple low-power devices. The classical \(^2\) asynchronous MAC was widely studied in the literature, see e.g. a paper [24] in which a capacity region of asynchronous Gaussian MAC was derived. We are interested in asynchronous unsourced random access. There are only a few papers devoted to this topic also known under the name asynchronous neighbor discovery [25, 26] which are based on compressive sensing approach.

In this paper we extend our methods for the case of unsourced asynchronous fading MAC. We continue the research started in [27] where a decoder operating in the frequency domain was proposed. By this trick the synchronization problem was converted to the phase shift estimation problem. The joint decoder from [1, 2] appeared to be unstable for the asynchronous case. Thus, a new approach paired with the successive cancellation algorithm from [28] was proposed. Here we give complete simulation results for the new decoder. As we are interested in energy efficiency we provide energy-per-bit (\(E_b/N_0\)) vs number of active users (\(K_a\)) curves and show this decoder to outperform joint decoder for both synchronous and asynchronous cases.

II. SYSTEM MODEL

There are \(K_{tot}\) users in the system but only \(K_a \ll K_{tot}\) users are active at each time instant. Communication proceeds in a frame-synchronized fashion, the length of each frame is \(n\). Each user has \(k\) bits to transmit within a frame. Within this paper we consider single antenna synchronous and asynchronous quasi-static Rayleigh fading MACs.

Let us start with the synchronous scenario. Consider a transmission in time domain. The synchronous quasi-static Rayleigh fading MAC (FMAC) model [2] can be described as follows

\[
Y^n = \sum_{i=1}^{K_a} H_i X^n_i + Z^n,
\]

where \(X_i \in \mathbb{C}^n\) is a codeword transmitted by the \(i\)-th active user, \(Z^n \sim \mathcal{CN}(0, I_n)\) are i.i.d. realizations of noise and \(H_i \sim \mathcal{CN}(0, 1)\) are the fading coefficients which are independent of \(\{X^n_i\}\) and \(Z^n\). We assume \(H_i\) to be unknown both at the transmitters and the receiver (no-CSI assumption).

In accordance to our assumptions all users utilize the same message set \([M] \triangleq \{1, \ldots, M\}\) and the same codebook \(\mathcal{C} = \{X(W)\}_{W=1}^{M}\). In order to send a message \(W_i\) the \(i\)-th user will use a codeword \(X_i = X(W_i)\). Recall in addition that \(\|X_i\|^2 \leq nP\).

It is clear that completely synchronous scenario is not possible in any practical implementation. Indeed, in a real life the receiver observes a superposition of delayed codewords (see Fig. 1). In what follows by \(\tau_i\) we denote the delay value for the \(i\)-th user. In order to recover the codewords we need to be able to deal with delays, which is not so easy if our frame is aligned in a time domain. For this case, we suggest to perform a transmission (align a frame) in a frequency domain and use OFDM scheme. Thus, each frame has a short duration in the time domain, but the overall signal to be received by base station becomes wideband. One knows that the delayed signal (by value \(\tau\)) suffers only from the phase shift after being aligned in the frequency domain. This is equivalent to multiplying each channel by a phase shift \(e^{j\omega \tau}\) corresponding to the subcarrier frequency \(f = \frac{\Delta f}{2}\). One can efficiently compensate this phase shift by means of cyclic prefix (CP). As soon as the channel uses are aligned in the frequency domain, \(n\) (the length of the frame) becomes the number of subcarriers. The time domain representation of transmitted signal of \(i\)-th user is

\[
x_i(t) = \sum_{l=1}^{n} X_i f e^{2\pi j f (\tau_i + \Delta f) t}, \quad t \in \left[0, \frac{1}{\Delta f}\right],
\]

where \(\Delta f\) is the carrier spacing and the symbol duration is \(1/\Delta f\). The \(x_i(t)\) are the time-domain samples with the sampling period equals to \(1/\Delta f\). The transmitted and delayed by \(\tau_i\) signal in terms of normalized delay \(\Delta_i = 2\pi \Delta f \tau_i\) can be expressed as

\[
x_i(t) = H_i \sum_{l=1}^{N} X_i e^{2\pi j \Delta f (\tau_i + \Delta f) t} e^{j\Delta_i (1 - \frac{t}{\Delta f})}.
\]

Thus, the asynchronous quasi-static Rayleigh fading MAC (AFMAC) model can be described as follows

\[
Y^n = \sum_{i=1}^{K_a} H_i S(\Delta_i) X^n_i + Z^n,
\]

where

\[
S(\Delta) = \text{diag} \left( e^{j\Delta}, e^{2j\Delta}, \ldots, e^{nj\Delta} \right),
\]

\(X_i \in \mathbb{C}^n, H_i \sim \mathcal{CN}(0, I_n), Z \sim \mathcal{CN}(0, I_n), \Delta_i \sim U[0, 2\pi]\). We note that the values of \(H_i\) and \(\Delta_i\) are unknown to the receiver.

In the rest of the paper we drop the superscript \(n\) unless it is unclear. The decoder aims to recover the list of messages.
LDPC codebook and replace each 0 explain how to construct it. We start with a binary $[n_1, k]$ LDPC codebook and each user utilizes the same codebook. Let us denote it by $K$, which allows to decode messages in a slot. Without loss of generality let us consider the first slot. Recall that the number of users transmitting in the first slot. Let $K$, be the random variable, which denotes the number of users in a slot. Clearly,

$$\sum_{\nu=1}^{V} K_{\nu} = K, K_{\nu} \sim \text{Binom}(K, 1/V).$$

**Remark 1.** In what follows we assume fading coefficients to remain constant (so-called channel coherence time) within a slot rather that a frame for both FMAC and AFMAC. Thus, the application of more sophisticated irregular repetition slotted ALOHA (IRSA) protocol [29] is not possible in our scenario.

### III. Decoding algorithm

In this section we present a TIN-SIC decoding algorithm, which allows to decode messages in a slot. Without loss of generality let us consider the first slot. Recall, that $K_1$ is a number of users transmitting in the first slot. Recall that the users utilize the same codebook. Let us denote it by $C$ and explain how to construct it. We start with a binary $[n_1, k]$ LDPC codebook and replace each 0 with $+\sqrt{P}$ and each 1 with $-\sqrt{P}$.

Let us note that the joint decoder from [2] can be applied for this problem as well. The only difference is that we have more latent variables: $H = (H_1, H_2, \ldots, H_{K_1})$ and $\Delta = (\Delta_1, \Delta_2, \ldots, \Delta_{K_1})$. Based on our observations the typical behaviour of the joint decoder for FMAC is as follows. It first decodes the user with the highest received power. After detecting the strongest user, the joint decoder switches to a weaker one (perform soft or probabilistic subtraction) until all users have been successfully decoded. At the same time the joint decoder is highly unstable for AFMAC. It decodes the codeword of the strongest user with high probability but then cannot subtract it from the received vector and fails with decoding of the second strongest user. In order to explain this fact we note that even a small error in $\Delta$ leads to a large error in $e^{m\Delta}$ when $m$ is large (the latter value has a phase which is practically random). Thus, we decided to switch from the joint decoder to TIN-SIC decoder and improve the subtraction rule. Let us describe the main steps in more details. We need to introduce a Gaussian likelihood function

$$p(Y | \mathbf{X}, H, \Delta)$$

$$= \left( \frac{1}{\sqrt{\pi}} \right)^{2n_1} \exp \left( -\left\| Y - \sum_{i=1}^{K_1} H_i S(\Delta_i) X_i \right\|^2 \right),$$

where $\mathbf{X} = (X_1, X_2, \ldots, X_{K_1})$.

**A. TIN decoder**

The decoder consists of three steps performed for a single user with all other transmissions considered as noise. Let us consider these steps in more details.

1) Delay ($\Delta$) estimation: In order to estimate a normalized delay values $\Delta$ for the strongest user we calculate the following likelihood function (we can omit the user index as we are always considering the strongest one)

$$p(Y | \Delta) = \mathbb{E}_{\Delta} \left[ \prod_{i=1}^{n_1} p(Y_i | H, X_i, \Delta) \right]$$

We calculated this function by Monte Carlo sampling. We note that $p(Y | \Delta)$ has multiple local maximums, so at this step we create a list of possible $\Delta$ values.

2) Single user decoding: Given the delay list, the decoder proceeds to the next step. With the step we launch a single user decoder for each value of $\Delta$ from the list ($\Delta$ is fixed within this step). The main goal of this step is to recover a codeword and obtain an estimate of the fading coefficient ($H$) for the strongest user. The factor graph corresponding to the TIN decoder is presented in Fig. 2. The message passing algorithm corresponding to this factor graph consists of 4 types of messages. Let us consider the messages in more details ($l = 1, \ldots, n_1$).

a) **Message type 1**: The messages from functional nodes to fading node are calculated as follows

$$R_l(H) = \mathbb{E}_{\Delta} \left[ p(Y_l | H, X_l, \Delta) \right].$$

b) **Message type 2**: The messages from fading node to functional nodes are calculated as follows

$$Q_{i}(H) = \prod_{t=1, t \neq i}^{N} \left[ R_t(H) \right] p(H).$$

c) **Message type 3**: The messages from functional nodes to code nodes (log-likelihood ratios, LLRs) are calculated as follows

$$L(x_i) = \log \frac{\mathbb{E}_H \left[ p(Y_l | H \sqrt{P}) \right]}{\mathbb{E}_H \left[ p(Y_l - H \sqrt{P}) \right]},$$

Fig. 2. Iterative single user decoding algorithm, factor graph with user code $C$ and latent variable $H$. The delay value $\Delta$ is fixed.


\[ d) \text{Message type 4:} \ \text{Decode LDPC code and update LLRs.} \]

Analogous to [2] all the procedures above can be efficiently implemented with use of Gaussian mixture (GM) pdf approximation.

**B. Known codeword subtraction**

This section is devoted to the known codeword subtraction procedure from the received signal vector \( Y \) or the SIC (successive interference cancellation) procedure.

Consider the TIN decoder from the previous section as a function \( \mathbb{D} \) that returns some codeword or an empty set. The latter case means the decoding failure due to the CSI estimation failure (due to the randomness caused by Monte Carlo sampling embedded into decoder). Multiple decoding attempts can fix this problem. Thus, the maximum TIN-SIC attempt count should be greater than the intended number of users in a slot.

The task of the decoder is to return the set of unique codewords extracted from the signal mixture \( Y \) (see (2)). Let us denote this codeword set as \( C_0 \). During the decoding process this set will be updated as follows

\[ C_0 = C_0 \cup \mathbb{D}(Y'), \]

where \( Y' \) is a residual value of \( Y \).

To subtract the codeword set \( C_0 = \{c_1, \ldots, c_\ell\} \), from the received signal, the SIC algorithm performs the following operations

\[ \left( \hat{H}_1, \ldots, \hat{H}_\ell, \hat{\Delta}_1, \ldots, \hat{\Delta}_\ell \right) = \arg \min_{(H_1, \ldots, H_\ell, \Delta_1, \ldots, \Delta_\ell)} \left\| Y - \sum_{\ell=1}^\ell H_\ell S(\Delta_\ell) c_\ell \right\| \]

and

\[ Y' = Y - \sum_{\ell=1}^\ell \hat{H}_\ell S(\hat{\Delta}_\ell) c_\ell. \]

This is algorithm is inspired by a well-known orthogonal matching pursuit (OMP) approach described in [28]. In our case, the space defined by successfully decoded codewords is parametrized by \( \{\hat{H}_i\} \) and \( \{\hat{\Delta}_i\} \). The gradient descent over these parameters helps to find a local minimum, and a grid search over \( \hat{\Delta}_i \) helps to find a global minimum. We perform this initial grid search separately for every successfully decoded user. The formal algorithm description is given by Algorithm 1.

**Algorithm 1 TIN-SIC decoder for AFMAC.**

\[
\begin{align*}
C_0 & \leftarrow \emptyset \\
Y' & \leftarrow Y \\
\text{for } a = 1, \ldots, A_{\text{max}} & \text{ do} \quad \text{Run } A_{\text{max}} \text{ decoding attempts} \\
& \quad \text{Construct the delay list } \Delta \\
& \quad \text{for } i = 1, \ldots, |\Delta| \text{ do} \quad \text{Try multiple } \Delta \text{ estimates} \\
& \quad \quad C' = \mathbb{D}(Y', \Delta_i) \quad \text{Perform decoding attempt} \\
& \quad \quad C_0 = C_0 \cup C' \quad \text{Update the set of unique codewords} \\
\text{end for} \\
& \quad \text{(} \hat{H}_1, \ldots, \hat{H}_\ell, \hat{\Delta}_1, \ldots, \hat{\Delta}_\ell \text{)} \\
& \quad Y' = Y - \sum_{\ell=1}^\ell \hat{H}_\ell S(\hat{\Delta}_\ell) c_\ell \quad \text{Perform cancellation} \\
\text{end for}
\end{align*}
\]

![Fig. 3. TIN-SIC synchronous scenario. Simulation results for a slot decoding performance for different \( K_1 \) (the number of simultaneous transmissions within a single slot). Transmission scheme: LDPC code with \( k = 100 \) information bits, \( R = 1/4 \) and BPSK modulation.](image)

**IV. NUMERICAL RESULTS**

Let us start with the results for a slot decoding. Recall that \( K_1 \) is the number of active users within a slot and \( P_e \) is a per-user probability of error. \( P_e \) as a function of \( E_b/N_0 \) is presented in Fig. 3 for the synchronous channel (1) and in Fig. 4 for the asynchronous channel (2). As one can see, the decoder can extract high number of messages from a single slot in both synchronous and asynchronous cases. Due to complexity reasons we limited our study in the joint decoding algorithm [2] by \( T = 4 \), i.e. the decoder was able to extract up to 4 messages from the slot. But in the TIN-SIC case, even 5 or 6 messages can be decoded simultaneously with relatively low \( P_e \). We note that a cyclic prefix length affects the energy efficiency. We ignored this \( E_b/N_0 \) shift in Fig. 4.

The final \( T \)-fold ALOHA performance (we measured the minimal \( E_b/N_0 \) value that guarantees the PUPE to be less than \( \epsilon \) for different number of active users \( K_a \) is presented in Fig. 5. One can see that TIN-SIC scheme outperforms the joint decoder significantly for large \( K_a \). The difference between synchronous and asynchronous cases remains small (recall that we have ignored the CP length in \( E_b/N_0 \) calculation). As one can observe, new TIN-SIC scheme outperforms the genie-aided achievable bound based on 4-fold ALOHA.

As soon as the complexity of TIN-SIC decoder does not depend on the number of active users in a slot, one can easily increase the value \( T \) (by \( T \) we denote the maximal collision order which we try to resolve). We set \( T = 8 \) in all our simulations as well as we assumed \( P_e = 1 \) for all slots with
The number of active users in the system. The future work will focus on increasing the network capacity (in terms of the number of users with low PUPE) or reducing the energy consumption. The gap between previously demonstrated schemes depends on the total number of active users. We also suppose that this case still can be resolved with $P_e < 1$ by TIN-SIC decoder with $T = 8$ (like detecting some number of the strongest users).

**V. Conclusions and Future Work**

TIN-SIC architecture for the synchronous and asynchronous fading MACs has been presented. New approach allows to increase the network capacity (in terms of the number of users with low PUPE) or reduce the energy consumption. The gap between previously demonstrated schemes depends on the total number of active users in the system. The future work will be dedicated to LDPC code construction for such a scheme by means of properly designed density evolution method and achievability bounds derivation.

**References**


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