Remarks on massive random access

Yury Polyanskiy

Department of EECS
MIT

yp@mit.edu

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100 bits (data) 30000 real degrees of freedom

code

30000 real degrees of freedom

1 30000
Preview: what this talk is about?

100 bits (data) 30000 real degrees of freedom

same code

1 30000

... 

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Remarks on massive random access
100 bits (data) -> code -> 30000 real degrees of freedom

AWGN

same code

1 30000
Who cares about this model?
Random-Access vs MAC

UserID (data)

30000 real degrees of freedom

code

1 30000

code

AWGN

1 30000

same code
Random-Access vs MAC

UserID (data) 30000 real degrees of freedom

... same code

code

AWGN

UserID 30000 real degrees of freedom

... same code

genie

UserID 30000 real degrees of freedom

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1

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...
**Plan**

- **Question 1**: Is there a useful asymptotics to study?
- **Question 2**: What sparsification is more useful? (IDMA vs. Slotted ALOHA)
- **Question 3**: Limitations of linear codes.
Key definition: random-access (same codebook) code

Definition (P.’17)

\( f : [M] \rightarrow \mathbb{R}^n \) is a \((n, M, K_a, \epsilon)\) random-access code if \( \exists \) list-\(K_a\) decoder \(g\) s.t.

\[
\mathbb{P}[W_j \not\in g(f(W_1) + \cdots + f(W_{K_a}) + Z)] \leq \epsilon \quad \forall j \in [K_a]
\]

where \( W_i \overset{iid}{\sim} \text{Unif}[M] \). Fundamental limit \((E_b/N_0)\):

\[
E_b^*(n, M, K_a, \epsilon) = \frac{1}{2 \log_2 M} \min_f \sup_{j \in [M]} \|f(j)\|_2^2
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How to do asymptotics? Fixed \(K_a\) and \(n \to \infty\) is useless.

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How to do asymptotics? \(K_a = \mu n\) is impossible: \(K_a \gg M\)
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How to do asymptotics? Solution: \( K_a = \mu n \) and \( M = 2^k K_a \)
Same-codebook codes = compressed sensing

- random-access = all users share same codebook
- ... obviously decoding is upto permutation of users
- Equivalent to compressed-sensing [Jin-Kim-Rao'11]

\[
X = \begin{pmatrix} c_1 & \cdots & c_M \end{pmatrix}
\]

Let \( \beta \in \{0, 1\}^M \) with \( \beta_j = 1 \) if codeword \( j \) was transmitted

Then the problem is:

\[ Y = X\beta + Z, \]

Goal:

\[ \mathbb{E}[\|\beta - \hat{\beta}(Y)\|] \rightarrow \min \]

(linear regression with sparsity \( \|\beta\|_0 = K \) aka comp.sensing).

PUPE requirement translates to false-discovery rate (FDR) requirement:

\[ \|\hat{\beta}\|_0 \leq K. \]

Regime of \( K = \mu n, M = 2^k K \) and \( n \rightarrow \infty \): CS with fixed-aspect ratio sensing matrix.
Same-codebook codes = compressed sensing

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- Equivalent to compressed-sensing [Jin-Kim-Rao'11]
- Let same-codebook (column) vectors be $c_1, \ldots, c_j$.

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- Then the problem is:
  \[Y = X\beta + Z, \quad \text{Goal: } E[||\beta - \hat{\beta}(Y)||] \to \min\]
  (linear regression with sparsity \(||\beta||_0 = K_a\) aka comp.sensing).
- PUPE requirement translates to false-discovery rate (FDR) requirement: \(||\hat{\beta}||_0 \leq K_a\).
- Regime of \(K_a = \mu n, \ M = 2^k K_a\) and \(n \to \infty\): CS with fixed-aspect ratio sensing matrix.
• In [P.'ISIT-2017] a random-coding achievability bound was shown:

\[ E_b^*(n, M, K_a, \epsilon) \leq E_{rc}(n, M, K_a, \epsilon) \]

• The bound is messy, but let us study it numerically.
  ▶ Frame length \( n = 30000, 60000, 120000 \) (real d.o.f.)
  ▶ User payload: \( k = 100 \) bits
  ▶ Active users: \( K_a = 1 \ldots 1500 \) (variable)
  ▶ Target error PUPE = \( 10^{-3} \)
Comparing random coding bound at different $n$

Bounds do not look comparable.

Next: Normalize to $\mu = K a n$ – user density

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Comparing random coding bound at different $n$

The graph shows the comparison of random coding bounds at different $n$ values. The x-axis represents the number of active users, $K_n$, and the y-axis represents $Eb/No$, dB.

- The black line represents $n=30000, k=100$.
- The blue line represents $n=60000, k=100$.

The graph indicates that the bounds do not look comparable.
Comparing random coding bound at different $n$

Bounds do not look comparable.

Next: Normalize to $\mu = K_a n$ – user density
Comparing random coding bound at different $n$

Bounds do not look comparable.

Next: Normalize to $\mu = \frac{K_a}{n} \text{ – user density}$
Comparing random coding bound at different $n$ (user density)

![Graph showing the comparison of random coding bounds for different user densities $n$. The graph demonstrates how the Eb/No, dB varies with user density $\mu$ for different values of $n$.]

- $n=30000, k=100$
- $n=60000, k=100$
- $n=120000, k=100$

Alignment is much better, but still.

Next: Normalize to effective # of bits
Comparing random coding bound at different $n$ (user density)

Alignment is much better, but still. 
**Next:** Normalize to effective # of bits
Effective number of bits

- **Problem:** Consider two values of blocklength $n_1 < n_2$.
- ... with the same $k$ and $\mu$ we have $K_{a,1} = \mu n_1 < K_{a,2} = \mu n_2$.
- ... So comparison of $E_b/N_0$ is not quite fair.
• **Problem:** Consider two values of blocklength $n_1 < n_2$.

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• ... So comparison of $E_b/N_0$ is not quite fair.

• Let us introduce **effective number of bits** as

\[
k_{eff} = \log_2 \frac{M}{K_a}
\]

• ... and then effective $E_b/N_0$ becomes

\[
\left( \frac{E_b}{N_0} \right)_{eff} = \frac{1}{2k_{eff}} \sup_{j \in [M]} \| f(j) \|_2^2
\]
Comparing bounds at different $n$ (effective $E_b/N_0$)

We found the right scaling: $n$ almost does not matter.

Next: Take $n \to \infty$. 

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Comparing bounds at different $n$ (effective $E_b/N_0$)

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• We say that $\mathcal{E}$ is asymptotically achievable effective $E_b/N_0$ at $(M_{eff}, \mu, \epsilon)$ if $\exists (n, M, K_a, \epsilon)$ RA-code with $M = M_{eff}K_a$, $K_a = \mu n$ and codewords of energy

$$\|c\|_2^2 \leq 2\mathcal{E} \log_2 M_{eff}$$

for all $n \to \infty$.

• Asymptotic fundamental limit: minimal achievable $\mathcal{E}$, i.e.

$$E^*_\infty(M_{eff}, \mu, \epsilon) = \limsup_{n \to \infty} \frac{\log_2 M}{\log M_{eff}} E^*_b(n, M, K_a, \epsilon)$$
Recall connection to the compressed sensing.

Call $E > 0$ feasible at a given ratio $p/n$ and sparsity $\pi$ if:

$$Y = \sqrt{E}X\beta + Z, \quad Z \sim \mathcal{N}(0, I_n), \beta \in \mathbb{R}^p$$

- Columns of $X$ are of unit energy
- $\beta \in \{0, 1\}^p$ and $\|\beta\|_0 = \pi p$,
- $\exists \hat{\beta}(Y, X)$ such that

$$\|\hat{\beta}\|_0 \leq \mu n \quad \text{(FDR)}$$
$$\|\hat{\beta} - \beta\|_0 \leq 2\epsilon\|\beta\|_0$$

Then we have $E^*_\infty = \min \frac{E}{2\log_2 M_{eff}}$
• Recall connection to the compressed sensing.
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• Then we have $E^*_\infty = \min E \frac{E}{2 \log_2 M_{eff}}$

• When $X \overset{iid}{\sim} \mathcal{N}(0, 1/n)$ this is well studied in stat. physics.
Replica method prediction

• Consider a scalar problem:
  \[ B = \sqrt{E_1}A + N, \quad A \sim \text{Ber}(\pi) \perp N \sim \mathcal{N}(0, 1) \]

• Define \( I_1(E_1) = I(A; B) \) and

\[
p^*(E_1, \pi) = \min_{\hat{A}} \left\{ \mathbb{P}[A = 0 | \hat{A} = 1] : \mathbb{P}[\hat{A} = 1] = \pi \right\}
\]

• It can be seen that \( p^* \) is a solution of

\[
\sqrt{E_1} = Q^{-1}(p^*) + Q^{-1}\left(\frac{\pi p^*}{1 - \pi}\right).
\]
Replica method prediction

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- Stat. physics predicts that inference in
  \[ Y = \sqrt{E}X\beta + Z, \quad X \overset{iid}{\sim} \mathcal{N}(0, 1/n), \beta \sim \text{Ber}^{\otimes p}(\pi) \]
  is asymptotically equivalent to a scalar problem with \( E_1 = E\eta \)

- \( \eta \in [0, 1] \) (the multi-user efficiency) is given as a solution of
  \[ \eta = \arg\min_x \left[ \frac{p}{n}I_1(xE) + \frac{1}{2}(x - 1 - \ln x) \right] \]
\[ B = \sqrt{\eta E}A + N, \quad A \sim \text{Ber}(\pi) \perp N \sim \mathcal{N}(0, 1) \]

\[ Y = \sqrt{E}X\beta + Z, \quad X \overset{iid}{\sim} \mathcal{N}(0, 1/n), \beta \sim \text{Ber}^\otimes p(\pi) \]

**Theorem (Replica formula exact for binary \( \beta \))**

Consider a sequence of random variables

\[ V_n = \mathbb{P}[\beta_1 = 1|Y, X] \in [0, 1] \]

as \( p, n \to \infty \) with \( p/n = \text{const.} \). Then

\[ V_n \xrightarrow{(d)} \mathbb{P}[A = 1|B]. \]
\[ B = \sqrt{\eta E} A + N, \quad A \sim \text{Ber}(\pi) \quad \perp \quad N \sim \mathcal{N}(0, 1) \]
\[ Y = \sqrt{E} X \beta + Z, \quad X \overset{iid}{\sim} \mathcal{N}(0, 1/n), \beta \sim \text{Ber}^{\otimes p}(\pi) \]

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\[ V_n \overset{(d)}{\to} \mathbb{P}[A = 1|B]. \]

- Pfister-Reeves and Barbier-Macris have shown that
  \[ \text{Var}[\beta_1|Y, X] \to \text{Var}[A|B] \]
- This is not enough to conclude the proof.
\[ B = \sqrt{\eta E} A + N, \quad A \sim \text{Ber}(\pi) \perp N \sim \mathcal{N}(0, 1) \]
\[ Y = \sqrt{E} X \beta + Z, \quad X \overset{iid}{\sim} \mathcal{N}(0, 1/n), \beta \sim \text{Ber}^{\otimes p}(\pi) \]

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\[ V_n \overset{(d)}{\to} \mathbb{P}[A = 1|B]. \]

- Possible to argue indirectly for binary \( \beta \) only.
- If we have some sequence \( G_n = G_n(Y, X) \in [0, 1] \) s.t.
  \[ \mathbb{E}[(G_n - \beta_1)^2] \to \text{Var}[\beta_1|Y, X] \] then \( G_n \overset{(d)}{\to} \mathbb{E}[\beta_1|Y, X]. \)
  For binary, this is \( = \mathbb{P}[\beta_1 = 1|X, Y]. \)
- AMP started at true \( \beta \) yields such a \( G_n \). The law of \( G_n \) is known to converge to \( \mathbb{P}[A = 1|B]. \)
Finite blocklength bound vs. $n = \infty$ asymptotics

Lesson: The $P.'17$ bound is not tight.

Details: for each $K_a$ compute $E_\infty$ at $k_{eff} = k - \log_2(K_a)$.

Scale $E_\infty$ down by $k_{eff}$.
Lesson: The [P.’17] bound is $0.5 - 0.7$ dB not tight.
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Details: for each $K_a$ compute $E_\infty$ at

$$k_{eff} = k - \log_2(K_a).$$

Scale $E_\infty$ down by $\frac{k_{eff}}{k}$.
Issue with the asymptotics

- Existence of \((n, M, K_a, \epsilon)\) does not imply existence of \((Ln, LM, LK_a, \epsilon)\) code (with the same effective \(E_b/N_0\)).
- However, existence of \((n, M, \text{Poi}(K_a), \epsilon)\)-code does imply the above as \(L \to \infty\).
- Let us give a formal definition.
RA-codes for random $K_a$

Channel model:

$$Y = f(W_1) + \cdots + f(W_T) + Z, \quad Z \sim \mathcal{N}(0, I_n), T \sim \text{Poi}(K_a)$$

where $W_i \overset{iid}{\sim} \text{Unif}[M]$.

**Definition**

$f : [M] \to \mathbb{R}^n$ is a $(n, M, \text{Poi}(K_a), \epsilon)$ random-access code if $\exists$ list-decoder $g$ s.t.

\[
\begin{align*}
\mathbb{E}[\# \{ j : W_j \notin g(Y) \}] & \leq \epsilon \mathbb{E}[T] \\
\mathbb{E}[|g(Y)|] & \leq \mathbb{E}[T]
\end{align*}
\]

Open question: There is no random coding bound at present.
Comparing $T$-fold ALOHA vs. IDMA

- Both $T$-fold Slotted ALOHA and IDMA try to sparsify collisions.
- $T$-SA partitions $n$ as $n = Ln_1$ where $L = K_a/T$. Collisions are in $n_1$-blocks.
- Asymptotic fundamental limit of $T$-SA is the minimal (effective) $E_b/N_0$ of a $(T/\mu, M_{\text{eff}}, \text{Poi}(T), \epsilon)$-codes.
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- Asymptotic fundamental limit of $T$-SA is the minimal (effective) $E_b/N_0$ of a $(T/\mu, M_{eff}, \text{Poi}(T), \epsilon)$-codes.
- Asymptotic fundamental limit of IDMA: Need to solve a (new?) sparse-design compressed sensing problem:

$$Y = \sqrt{E}X\beta + Z$$

- $X \in \mathbb{R}^{n \times p}$ has unit-energy columns.
- $X$ has constant $s = M_{eff}T$ non-zeros per-row!
- $p/n = \text{const}$, $E = \text{const}$, $n \to \infty$.
- **Open question:** Replica/AMP for $X \stackrel{iid}{\sim} \frac{c}{p} \mathcal{N}(0, c') + (1 - \frac{c}{p})\delta_0$
Limitations of linear codes
Binary Adder Channel (BAC)

\[ Y = A + B \quad A, B \in \{0, 1\}, Y \in \{0, 1, 2\} \]

- Maximal symmetric rate:
  \[ C_{sym} = \frac{1}{2} \max_{A, B} I(A, B; Y) = \frac{1}{2} \max H(A + B) = \frac{3}{4} \]

- Maximal same-codebook rate: \( C_{same} = \frac{3}{4} \)

- Maximal 0-error same-codebook rate: \( C_{0, same} \leq 0.5753 \) [Cohen-Litsyn-Zemor'01]

- Maximal 0-error symmetric rate: \( C_{0, sym} \geq 0.659 \) [Bross-Blake'98]
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- Thus, for low error-rate there is a penalty due to random access.
Limitations of linear codes

**Theorem (P.'19, unpublished)**

For any same-codebook $C \subset \{0, 1\}^n$ with $|C| = 2^k$, which is $\mathbb{F}_2$-affine. Then

$$P_e \geq \frac{1}{2} \left( 1 - \frac{n}{2k - 2} \right)$$

Thus, maximal achievable rate via such codes is $\leq \frac{1}{2}$.

- WLOG, assume no constant coordinates. Then average codeword weight $= \frac{n}{2}$.
- Consider two codewords $c_1, c_2 \in C$. Let $S = \{j : c_{1,j} = c_{2,j}\}$.
- If $|S| < k$ then $\exists 0 \neq c \in C$ with $c|_S = 0$. (since we have $n - k + |S| < n$ equations on $c$).
- Then $c_1 \oplus c$, $c_2 \oplus c$ is a confusing pair: it has the same real-sum.
- Bound $\mathbb{P}[|S| < k] = \mathbb{P}[|c_1 \oplus c_2|_H > n - k]$ via Chebyshev.
Adder MAC: open issues

\[ Y = A + B \quad A, B \in \{0, 1\}, Y \in \{0, 1, 2\} \]

- **Challenge:** Find same-codebook code beating 1/2 barrier.
Adder MAC: open issues

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- **Challenge:** Find same-codebook code beating 1/2 barrier.
- **Note:** two-phase schemes are not allowed
  - use first \(k_1\) data bits (out of \(k = nR \gg 1\)) to select a permutation matrix for each user.
  - Encode \(k_1\) via any low-rate code into first \(n_1\) coordinates.
  - Then use non-same LDPC codes on the rest \(n - n_1\) coordinates.
- **Why not allow?** e.g. the first \(k_1\) bits have degree \(\Omega(n)\) (i.e. affect all coded bits). Bad!
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- **Why not allow?** e.g. the first \( k_1 \) bits have degree \( \Omega(n) \) (i.e. affect all coded bits). Bad!
- **Challenge:** How to decode sparse-graph same-codebook codes?
- **Note:** the local beliefs about bits are useless. Need to introduce global “super nodes”.

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• Question 1: Is there a useful asymptotics to study?
  A: Yes $K_a = \mu n$, $M = 2^{k_{eff}} K_a$, $n \to \infty$.

• Question 2: What sparsification is more useful? (IDMA vs. $T$-SA)
  A: Need to solve $Poi(K_a)$ problem. Need to solve sparse-design CS.

• Question 3: Limitations of linear codes.
  A: Non-capacity achieving. **Challenge:** Find alternatives.
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Thank you!